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Crack growth resistance curve and size effect in the fracture of cement paste

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A general theory is presented for the fracture of cementitious materials. It is shown that crack growth resistance curves can be constructed for cement pastes using fracture data available in the literature. The crack growth resistance curves are used to explain the specimen size and crack length dependence of fracture toughness in cement pastes.

1. Introduction

Although the fundamental concepts of linear elastic fracture mechanics (LEFM) have been successfully applied to a wide range of engineering materials there is considerable disagreement as to whether these fracture concepts can be used to determine the fracture properties of cement paste, mortar and concrete. The main concern is that unless very large specimens are employed the fracture toughness (K_c) values obtained for these cementitious materials are size-dependent and are invalid fracture measurements [1, 2]. Even when the fracture process zone is comparatively small as in the case of cement paste, the experimental results [1, 3–11] are quite often contradictory. While some investigators [3–9] have shown that the strength and flaw size of cement paste are related by the K_c concept, others [1, 10, 11] dissented from this view and showed that K_c either increases with size [1, 10] or decreases with crack length [11] so that LEFM concepts are inapplicable.

It must, however, be pointed out that even if K_c is dependent on specimen size or crack length, this does not necessarily mean that LEFM is invalid. In this paper, we propose that the crack growth resistance (K_R) curve concept developed within the framework of LEFM may be used to explain the size-dependent K_c results of cement paste. The crack growth resistance curve concept was first used to explain size effects in thin sheet metals under essentially plane stress conditions. The shear lip formation in metals associated with plane stress gives rise to an increase in K_R with crack growth. K_R curves for both mortar and concrete have already been established by Wecharatana and Shah [12]. The fact that cement paste has a rising K_R curve with crack extension (Δa), i.e. stable crack growth, before final instability is not new. Such a phenomenon has been recorded by several investigators [6, 13–15]. It may be suggested that the origins of stable crack growth are partly due to the pull-out of calcium silicate hydrate (CSH) fibrils behind the crack tip [1, 16] and partly due to the cement grains or unbroken short segments bridging across the crack faces [13]. The increased work required to cause separation of the CSH fibrils and the bridging segments in order to keep the crack tip advancing must give rise to the K_R-curve effect. Microcracking in the wake of the crack tip may also cause toughening as the crack extends. The zone over which these toughening mechanisms occurred may be conveniently called the fracture process zone.

The "fictitious crack" model of Hillerborg and coworkers [17, 18] can be used to analyse the fracture size dependence of cement paste. In this method the fracture process zone is replaced by a fictitious crack which can still transmit a stress. The transmitted stress at any point along the fictitious crack is a function of the crack opening at that point which can be obtained from stress–displacement measurements made during a tensile test. At the tip of the fictitious crack the stress is finite and equal to the maximum tensile strength. In a stable specimen, such as a displacement-controlled double cantilever, the fictitious crack grows under increasing load until the stress transmitted at the tip of the true crack drops to zero and the crack tip opening displacement (CTOD) reaches its maximum value, $(\mathrm{CTOD})_c$. During this stage the true crack remains stationary. If the load is further increased the true crack begins to grow as well as the fictitious crack. The main problem with the Hillerborg model is that it requires considerable computation time. Hillerborg has used the finite element method in his analysis, though we have shown that the displacements can be found from the standard stress intensity solutions by the use of Castigliano's method [19].

Jenq and Shah [20] have proposed a simpler two-parameter method to account for size dependence in cementitious materials. However, we believe that their method, though it has been used to achieve remarkable agreement with the data obtained by Higgins and Bailey [1] for cement paste, is physically unsound. In the Jenq and Shah model the fracture process zone is replaced by a fictitious crack as in the Hillerborg model. They assume that for geometries like the notch bend specimen, where for constant load the stress intensity factor due to the applied load increases with crack length, the maximum load and hence the critical K_c are obtained when two material parameters reach critical values. These parameters are: (i) the stress

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intensity factor at the tip of the fictitious crack due to the applied loads, and (ii) the CTOD at the tip of the true crack due to the applied loads. We shall show that the first condition, that unstable crack growth occurs at a critical stress intensity factor K^{s}_{Ic}, is a reasonable assumption for notch bend specimens. However, the actual CTOD at the tip of the true crack is the sum of the CTOD due to the applied loads and that due to the stresses transmitted across the fictitious crack. This latter component of the CTOD is not insignificant and cannot be neglected. Furthermore, we shall show that in the notch bend geometry the specimen becomes unstable before the fracture process zone is fully developed and that the CTOD is less than the critical value.

In this paper we apply our development of crack growth resistance curves for fibre-reinforced materials [19, 21–24] to the analysis of cement paste.

2. Fracture theory for cementitious materials

We assume that the fracture process zone where the material is no longer completely coherent can be modelled by Hillerborg's fictitious line crack (Fig. 1). Since the stress at the tip of the fracture process zone is finite and for practical purposes equal to the maximum tensile strength of the material, there can be no singularity at the tip of the fictitious crack. Hence the effective stress intensity factor K at the tip of the fictitious crack, which is the sum of the stress intensity factor K_a due to the applied loads and K_r due to the stresses in the fracture process zone, must be zero, i.e.

$$K = K_a + K_r = 0 \qquad (1)$$

The crack growth resistance K_R is defined as the applied stress intensity factor K_a and therefore

$$K_R = -K_r. \qquad (2)$$

This expression is positive since the stresses in the fracture process zone tend to close the crack and hence produce a negative stress intensity factor K_r which

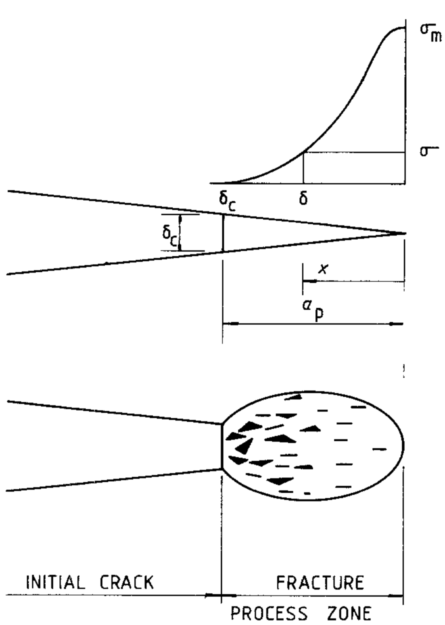

Figure 1 Fracture model for cementitious materials.

can be numerically calculated for a given specimen geometry [19].

In common with Jenq and Shah [20] we propose that the true crack will not propagate until the CTOD at its tip reaches a critical value δ_c. However, we use the total CTOD (δ) given by

$$\delta = \delta_a + \delta_r \qquad (3)$$

where δ_a is the CTOD due to the applied loads and δ_r is the negative CTOD due to the stress transmitted across the fictitious crack, which can be calculated from Castigliano's method [19]. Like Hillerborg and co-workers [17, 18] we assume that the stress–displacement relationship for the fracture process zone is unique and a material property. So far our method is identical to that of Hillerborg but our solution is far simpler.

We have shown [19] that a reasonable approximation to K_R curves can be obtained for cementitious materials if it is assumed that the crack faces remain straight as they deform. Furthermore we have shown that it is not necessary to model the stress–displacement curve precisely and that a linear stress–displacement relationship can be used [19]. The same approximations are used in this paper. Our equivalent linear stress–displacement relationship for a fully developed fracture process zone of length a_p is given by

$$\sigma = \sigma_m \left(1 - \frac{\delta}{\delta_c}\right) = \sigma_m \left(1 - \frac{x}{a_p}\right) \qquad (4)$$

where x is measured as in Fig. 1. To obtain a K_R curve we first calculate the fully developed fracture process zone size a_p from the conditions that $K = 0$ at the fictitious crack tip and the CTOD is the critical value δ_c, and noting that the stress in the process zone is given by Equation 4. This calculation requires iteration but converges very rapidly compared to an exact solution where the complete shape of the process zone has to be found by iteration [19]. For loads that do not develop the full fracture process zone, we assume that Equation 4 still applies. The stresses in the process zone are then known and the K_R curve can be calculated from Equation 2.

In very large specimens the K_R curve reaches a plateau value K_∞ when the fracture process zone is fully developed, given by

$$K_\infty = (E\omega)^{1/2} \qquad (5)$$

where ω is the area under the stress–displacement curve for the process zone. However, in smaller specimens K_R will not reach a plateau and it is dependent on the specimen size and geometry. This dependence is particularly strong for the notch bend geometry where in small specimens K_R can be considerably greater than K_∞ [24].

In most geometries the specimen will become unstable at loads less than those required to develop a full fracture process zone. The critical stress intensity factor K_c at which instability occurs is obtained [23] from the conditions

$$K_c = K_a = K_R \qquad (6)$$

and

$$\left(\frac{\partial K_a}{\partial a}\right)_{K_a = K_c} = \frac{dK_R}{da} \qquad (7)$$

where a is the effective crack length referred to the tip of the fictitious crack. If the appropriate K_R curve is constructed from knowledge of σ_m and δ_c, the critical stress intensity factor can be found by iteration.

3. Crack growth resistance curves for cement paste and the size dependence of K_c

The most comprehensive data on the size dependence of K_c for cement pastes are those of Higgins and Bailey [1] for notch bend specimens. Watson [25] has collected many K_c data, but they cannot be analysed because there are different water/cement ratios which affect the K_c values [26]. We have used the method outlined in the previous section to construct K_R curves for the specimens of Higgins and Bailey from estimates of σ_m and $E\delta_c$. These K_R curves were then used to calculate the critical stress intensity factors K_c. Although Higgins and Bailey used three-point load bend specimens we have made our calculations assuming pure bending because we possessed a previous computer program [24] that could be very simply modified to solve the present problem. The difference for a given bending moment between the stress intensity factor for three-point loading and pure bending is slight and does not introduce a significant error. The critical stress intensity factors K_c are calculated for the *initial notch* length, as were the experimental values obtained by Higgins and Bailey. These values are significantly less than the values of the critical applied stress intensity factors at the tip of the fictitious crack obtained from Equations 6 and 7.

By varying σ_m and $E\delta_c$ we obtained the values of K_c that gave the best fit to the results of Higgins and Bailey [1] (Fig. 2). The actual values of K_c are not very sensitive to σ_m and $E\delta_c$ provided K_∞ was kept constant at about 0.72 MPa $m^{1/2}$. This insensitivity to the actual relationship used for the stress–displacement curve adds weight to our argument that any reasonable equivalent curve can be used in place of the actual one. Although the theoretical curves do not give as close a fit to the mean value of K_c for the crack size of Higgins and Bailey's results [1] as those of Jenq and Shah [20], they do demonstrate that the size effect on K_c is due to the crack growth resistance of a developing fracture process zone. Also the curves show that though K_c is relatively constant for a given size of specimen for a/d ratios (where d = specimen depth) of 0.1 to 0.5, there is a marked drop in K_c for a/d ratios of less than 0.1 as is indicated by the experimental data.

In Fig. 3, we show the K_R curves appropriate to the specimen size for an a/d ratio of 0.25. Only the two largest specimens have K_R curves that tend to a plateau value close to the theoretical value for an infinitely large specimen of 0.72 MPa $m^{1/2}$. Fig. 3 also shows that the size of the fully developed fracture process zone is dependent on the specimen size.

We have superimposed the locus of the instability criteria Equations 6 and 7 on the K_R curves presented in Fig. 3. Here the critical values of the stress intensity factor are those calculated at the tip of the fictitious crack and hence are larger than those given in Fig. 2. It is interesting to note that there is little variation in K_c calculated at the tip of the fictitious crack whereas K_c obtained at the tip of the initial notch is

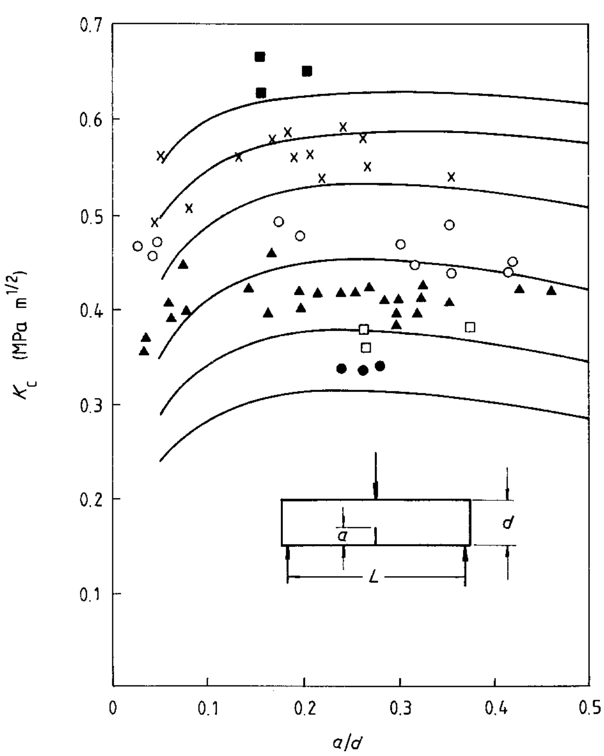

Figure 2 The critical stress intensity factor K_c as a function of initial notch length for various specimen sizes with $L = 5d$ (experimental data from Higgins and Bailey [1]). d (mm) = (●) 5, (□) 8, (▲) 14, (○) 28, (x) 56, (■) 110.

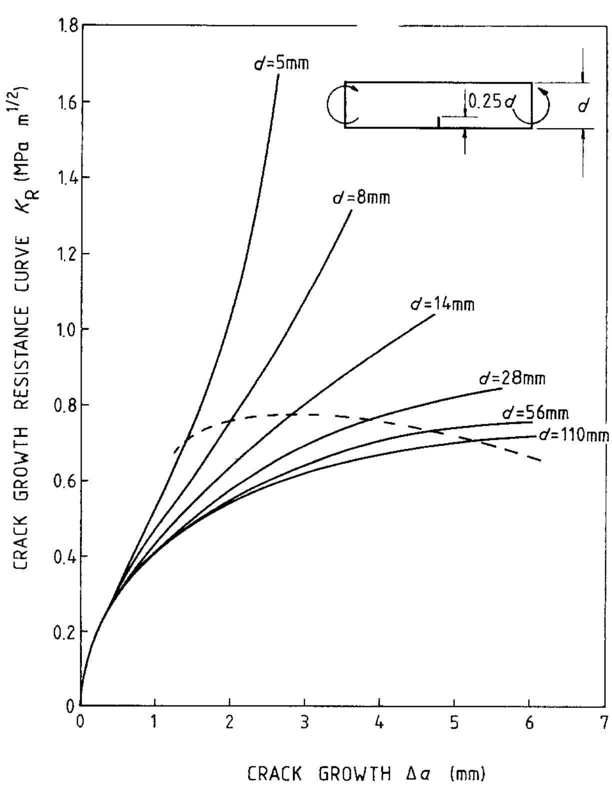

Figure 3 Crack growth resistance curves for various specimen sizes for a/d = 0.25. (– – –) Instability locus.

significantly dependent on the specimen size. Thus the assumption by Jenq and Shah [20] that K_{Ic}^{s} is a material constant is reasonably accurate for the notch bend geometry. However, since K_R curves for other geometries are significantly different [24] this assumption may not hold for all geometries.

Also at fracture instability, as Fig. 3 shows, the fracture process zone is not fully developed so that the CTOD at the true crack tip is less than δ_c. This result is in direct contrast to the Jenq and Shah assumption of a critical CTOD being satisfied at instability for the notch bend geometry.

It is interesting to compare the values of σ_m and $E\delta_c$ found empirically from our analysis (8 MPa and $0.13\,\mathrm{MN\,m^{-1}}$, respectively) with the estimates of 12 MPa and $0.03\,\mathrm{MN\,m^{-1}}$ given by Higgins and Bailey [1]. Although the agreement is not close they are of the same order, and since Higgins and Bailey used the Dugdale model for the process zone (i.e. constant stress, which is inappropriate for cementitious materials) their value of $E\delta_c$ should strictly be compared with half of our estimate because we believe that K_∞ is the more fundamental parameter.

4. Discussion

For a given specimen depth d, K_c is reasonably independent of crack length a provided that the ratio a/d is greater than about 0.1. Thus by plotting the fracture stress against the square root of the crack length, we can fit the data reasonably well with a constant K_c locus. Therefore many investigators working on single-size specimens have concluded that Griffith's equation is suitable for fracture analysis. Unfortunately K_c increases with specimen size as the data of Higgins and Bailey [1] and also Strange and Bryant [10] show. Thus we consider that there is sufficient evidence for a size effect on the fracture toughness of cement pastes. Although crack speeds can affect K_c in many materials [27, 28], judging from the slow crack growth data [15] it is doubtful whether this can be a significant factor in affecting K_c values of different specimen sizes. The crack growth resistance curve concept explains the size effects. Apart from the support given to this concept from the data of Higgins and Bailey [1] plotted in Fig. 2, the results from Brown and Pomeroy [6] for double-cantilever-beam specimens with a water/cement (w/c) ratio of 0.47 also point to the existence of a K_R curve. In these experiments K_c rises from about $0.25\,\mathrm{MPa\,m^{1/2}}$ to a plateau value of $0.40\,\mathrm{MPa\,m^{1/2}}$ as the crack grows by about 30 mm. A crack growth resistance can only occur if there is a significant fracture process zone where CSH fibrils and bridging segments carry some load. Measurements of the fracture process zone is difficult. Higgins and Bailey [14] have shown the existence of a fracture process zone of the order of a millimetre in extent, but there are no detailed measurements as a function of specimen size d that enable a comparison with our predictions of the exact size shown in Fig. 3.

5. Conclusion

A general theory has been presented for the fracture mechanics of cementitious materials. Using the fracture data of Higgins and Bailey it is possible to obtain crack growth resistance curves for cement pastes that can be used to explain size and crack length effects on fracture toughness. The crack growth resistance curve is only unique for very large size specimens. For smaller specimens the curves deviate as the fracture process zone develops, because this development significantly affects the compliance of the specimen.

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